# Lecture 20

In this lecture we continue our study of logic, focussing on truth tables. We also look briefly at the idea of two compound propositions being logically equivalent.

# Truth Tables

We have 5 connectives:

$\neg$ (not) — unary

$\wedge$ (and)
$\vee$ (or)
$\Rightarrow$ (implies)
$\Leftrightarrow$ (is equivalent to)

$\left.\begin{array}{l}\wedge \\ \vee \\ \Rightarrow \\ \Leftrightarrow\end{array}\right\}$ binary

We combine elementary (or constituent) propositions to create compound propositions.

The truth values of the constituent propositions determine the truth values of a compound proposition.

How this is done is described in the following truth tables.

<u>negation</u>

not

| $P$ | $\neg P$ |
|---|---|
| T | F |
| F | T |

## <u>conjunction</u>

and

| $P$ | $Q$ | $P \land Q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |

The compound propⁿ "P and Q" is only true in the case where P is true and Q is true.

## <u>disjunction</u>

or

| $P$ | $Q$ | $P \lor Q$ |
|---|---|---|
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

Note that "$P$ or $Q$" remains true if $P$ and $Q$ are both true. (This is the "inclusive" or.)

<u>implication</u>

| $P$ | $Q$ | $P \Rightarrow Q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

Note that the only implication which fails to be true is "true implies false".

So <u>every false statement implies every other statement, whether true or false.</u>

"If $P$ then $Q$" says that if $P$ is true then $Q$ is true. It doesn't say anything about what happens when $P$ is false. So if $P$ is false, we don't care whether $Q$ is true or false.

Note also that <u>every true statement is implied by every other statement, whether true or false.</u>

<u>equivalence</u>

| $P$ | $Q$ | $P \Leftrightarrow Q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | T |

Memorise these 5 truth tables.

From them, all other truth tables can be constructed.

<u>E.g.</u> Construct the truth table for the compound proposition $\neg P \vee Q$.

<u>Sol$^n$</u>

| $P$ | $Q$ | $\neg P$ | $\neg P \vee Q$ |
|---|---|---|---|
| T | T | F | T |
| T | F | F | F |
| F | T | T | T |
| F | F | T | T |

Same truth table as for implication.

So:

$$P \Rightarrow Q \equiv \neg P \vee Q$$

is logically equivalent to
(has the same meaning as)

<u>E.g.</u> $(P \Rightarrow Q) \wedge (Q \Rightarrow P)$

| $P$ | $Q$ | $P \Rightarrow Q$ | $Q \Rightarrow P$ | $(P \Rightarrow Q) \wedge (Q \Rightarrow P)$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | F | F | T | F |
| F | T | T | F | F |
| F | F | T | T | T |

Same as equivalence.

So:

$$P \Leftrightarrow Q \equiv (P \Rightarrow Q) \land (Q \Rightarrow P)$$